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Nonlinear conserved-charge coherent state and its relation to nonlinear entangled state

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Abstract

By suitably averaging over $U(1)$-group action on a two-mode nonlinear coherent state we derive the nonlinear conserved-charge coherent state. Based on it, a new nonlinear entangled state $|\eta\rangle_{f,g}$ and its deductive state are introduced. Their completeness relation is proved. A new two-mode nonlinear squeezing operator in $|\eta\rangle_{f,g}$ representation is obtained, which in turn leads to a nonlinear negative binomial state.

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1. Introduction

Many quantum states of a radiation field, such as a squeezed state, phase states, negative binomial state, can be viewed as a sort of nonlinear coherent state (NCS) [1–9]. NCS has been paid a great deal of attention in recent years by physicists. The most interesting aspect is that they exhibit nonclassical features [10–13]. A class of NCS can be realized physically as the stationary states of centre-of-mass motion of a trapped ion [14]. The single-mode NCS is defined as

$$f(N_a)a\,|\alpha\rangle_f = \alpha\,|\alpha\rangle_f \tag{1}$$

where the subscript f means that the eigenstate $|\alpha\rangle_f$ is related to the operator $f(N_a)$, $N_a = a^\dagger a$. By considering the relations

$$\begin{gathered} N_a a = a(N_a - 1) \qquad N_a a^\dagger = a^\dagger(N_a + 1) \\ \left[f(N_a)a, \frac{1}{f(N_a-1)}a^\dagger\right] = \left[a, a^\dagger\right] = 1 \end{gathered} \tag{2}$$

one can ensure that the single-mode NCS has the form

$$|\alpha\rangle_f = \exp\left[\frac{\alpha}{f(N_a-1)}a^\dagger\right]|0\rangle \tag{3}$$

where $a\,|0\rangle = 0$. By straightforwardly extending $|\alpha\rangle_f$ to the two-mode case we get

$$|\alpha, \beta\rangle_{f,g} = \exp\left[\frac{\alpha}{f(N_a-1)}a^\dagger + \frac{\beta}{g(N_b-1)}b^\dagger\right]|00\rangle = |\alpha\rangle_f \otimes |\beta\rangle_g\,. \tag{4}$$

In this paper, we want to extend the Bhaumik–Baumik–Dutta–Roys (BBDR) coherent states of charged bosons [15] to the nonlinear operator case, and construct nonlinear conserved-charge coherent states (NCCCS). The BBDR coherent states are the common eigenvector of $N_a - N_b$ and ab, since $[N_a - N_b, ab] = 0$:

$$ab\,|\alpha, q\rangle = \alpha\,|\alpha, q\rangle \qquad (N_a - N_b)\,|\alpha, q\rangle = q\,|\alpha, q\rangle \tag{5}$$

where

$$|\alpha, q\rangle = C_q \sum_{n=0}^{\infty} \frac{\alpha^n}{[n!(n+q)!]^{1/2}}\,|n+q, n\rangle \tag{6}$$

which, in quantum optics theory, is called a pair coherent state [16]. Here we observe

$$[N_a - N_b, f(N_a)ag(N_b)b] = 0$$

so we try to construct NCCCS, denoted as $\left|r^2, q\right\rangle_{f,g}$, which obeys the equations

$$\begin{aligned}(N_a - N_b)|r^2, q\rangle_{f,g} &= q|r^2, q\rangle_{f,g}\\ f(N_a)g(N_b)ab|r^2, q\rangle_{f,g} &= r^2|r^2, q\rangle_{f,g}.\end{aligned} \tag{7}$$

We then show that NCCCS is related to a kind of nonlinear entangled state, which makes up a completeness relation and is able to constitute a new nonlinear quantum mechanical representation. In this representation, a new two-mode nonlinear squeezing operator can be naturally introduced, and a nonlinear negative binomial state can then be defined. Through our discussions, the technique of integration within an ordered product of operators for nonlinear Bose operators is used.

2. Nonlinear conserved-charge coherent state and its relation to nonlinear entangled state

We now derive NCCCS in terms of $|\alpha, \beta\rangle_{f,g}$ by setting

$$\alpha = \lambda \mathrm{e}^{-\mathrm{i}(\theta+\varphi)} \qquad \beta = \mu \mathrm{e}^{-\mathrm{i}(\theta-\varphi)}$$

and then making an average for $|\alpha, \beta\rangle_{f,g}$ over a $U(1)$ phase $\mathrm{e}^{\mathrm{i}q\varphi}$, i.e.

$$\begin{aligned}\left|\lambda\mu \mathrm{e}^{-2\mathrm{i}\theta}, q\right\rangle_{f,g} &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi\ \mathrm{e}^{\mathrm{i}q\varphi}\,|\alpha, \beta\rangle_{f,g}\\ &= N_q \sum_{n=0}^{\infty} \frac{\left(\lambda\mu \mathrm{e}^{-2\mathrm{i}\theta}\right)^n}{[n!(n+q)!]^{1/2}\prod_{l=0}^{n+q-1} f(l)\prod_{m=0}^{n-1} g(m)}\,|n+q, n\rangle\end{aligned} \tag{8}$$

where N_q is a normalization constant depending on f and g. Let $\lambda = \mu = r\mathrm{e}^{\mathrm{i}\theta}$. Then (8) becomes

$$\begin{aligned}\left|r^2, q\right\rangle_{f,g} &= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi\ \mathrm{e}^{\mathrm{i}q\varphi}\exp\left[\frac{r\mathrm{e}^{-\mathrm{i}\varphi}}{f(N_a-1)}a^\dagger + \frac{r\mathrm{e}^{\mathrm{i}\varphi}}{g(N_b-1)}b^\dagger\right]|00\rangle\\ &= N_q \sum_{n=0}^{\infty} \frac{r^{2n}}{[n!(n+q)!]^{1/2}\prod_{l=0}^{n+q-1} f(l)\prod_{m=0}^{n-1} g(m)}\,|n+q, n\rangle\,.\end{aligned} \tag{9}$$

One can check that the NCCCS $\left|r^2, q\right\rangle_{f,g}$ is really the common eigenstate of $f(N_a)g(N_b)ab$ and $N_a - N_b$. When $f = g = 1$, it reduces to the BBDR charged coherent state (6). Furthermore, due to

$$\left[\frac{1}{f(N_a-1)g(N_b-1)}a^\dagger b^\dagger, N_a - N_b\right] = 0 \tag{10}$$

we see that there exists another state which also conserves charge. In fact, by $\exp[\frac{1}{f(N_a-1)g(N_b-1)}a^\dagger b^\dagger]$ acting on $\left|r^2, q\right\rangle_{f,g}$ we have

$$\begin{aligned} &\mathrm{e}^{-r^2/2-1/[f(N_a-1)g(N_b-1)]a^\dagger b^\dagger}\left|r^2, q\right\rangle_{f,g} \\ &\quad= \frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{e}^{\mathrm{i}q\varphi}\exp\left[-r^2/2 - \frac{1}{f(N_a-1)g(N_b-1)}a^\dagger b^\dagger\right. \\ &\qquad\left. + \frac{r\mathrm{e}^{-\mathrm{i}\varphi}}{f(N_a-1)}a^\dagger + \frac{r\mathrm{e}^{\mathrm{i}\varphi}}{g(N_b-1)}b^\dagger\right]|00\rangle \equiv ||r^2, q\rangle_{f,g} \end{aligned} \tag{11}$$

where $||r^2, q\rangle_{f,g}$ obeys

$$(N_a - N_b)\left||r^2, q\right\rangle_{f,g} = q\left||r^2, q\right\rangle_{f,g}. \tag{12}$$

On the other hand, from (11) we can define a new state

$$\begin{aligned} |\eta \equiv r\mathrm{e}^{-\mathrm{i}\varphi}\rangle_{f,g} = \exp&\left[-|\eta|^2/2 - \frac{1}{f(N_a-1)g(N_b-1)}a^\dagger b^\dagger\right. \\ &\left. + \frac{\eta}{f(N_a-1)}a^\dagger + \frac{\eta^*}{g(N_b-1)}b^\dagger\right]|00\rangle. \end{aligned} \tag{13}$$

Then (11) is simplified as

$$\left||r^2, q\right\rangle_{f,g} = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\varphi\,\mathrm{e}^{\mathrm{i}q\varphi}\left|\eta = r\mathrm{e}^{-\mathrm{i}\varphi}\right\rangle. \tag{14}$$

The state $|\eta = r\mathrm{e}^{-\mathrm{i}\varphi}\rangle_{f,g}$ obeys

$$\begin{aligned} &\left[f(N_a)a + \frac{1}{g(N_b-1)}b^\dagger\right]|\eta\rangle_{f,g} = \eta\,|\eta\rangle_{f,g} \\ &\left[g(N_b)b + \frac{1}{f(N_a-1)}a^\dagger\right]|\eta\rangle_{f,g} = \eta^*\,|\eta\rangle_{f,g} \end{aligned} \tag{15}$$

and

$$\left[f(N_a)a + \frac{1}{g(N_b-1)}b^\dagger\right]\left[g(N_b)b + \frac{1}{f(N_a-1)}a^\dagger\right]|\eta\rangle_{f,g} = r^2\,|\eta\rangle_{f,g}. \tag{16}$$

It then follows from (14) and (16) that

$$\left[f(N_a)a + \frac{1}{g(N_b-1)}b^\dagger\right]\left[g(N_b)b + \frac{1}{f(N_a-1)}a^\dagger\right]\left||r^2, q\right\rangle_{f,g} = r^2\left||r^2, q\right\rangle_{f,g}. \tag{17}$$

Note that, though $||r^2, q\rangle_{f,g}$ also conserves charge, as shown by (12), it is not a coherent state (see the next section). When $f = g = 1$, $|\eta\rangle_{f,g}$ reduces to

$$|\eta\rangle = \exp\left[-|\eta|^2/2 - a^\dagger b^\dagger + \eta a^\dagger + \eta^* b^\dagger\right]|00\rangle$$

which is an entangled state constructed [17] according to the Einstein, Podolsky and Rosen (EPR) argument [18], since it is the common eigenvector of $x_1 + x_2 = \frac{1}{\sqrt{2}}\left(a + a^\dagger + b + b^\dagger\right)$ and $P_1 - P_2 = \frac{1}{\sqrt{2}\mathrm{i}}\left(a - a^\dagger - b + b^\dagger\right)$. Thus we call $|\eta\rangle_{f,g}$ the nonlinear entangled state. Now we conclude that $\left||r^2, q\right\rangle_{f,g}$ is a deductive state from the nonlinear entangled state.

3. IWOP technique for nonlinear Bose operators

For discussing the completeness relation of the nonlinear entangled states we appeal to the technique of integration within an ordered product (IWOP) for nonlinear operators. To begin with, let us first recall the IWOP technique for fundamental Bose operators $a^\dagger$ and a [19–24]:

(1) The order of Bose operators a and $a^\dagger$ within a normally ordered product : : can be permuted. That is to say, even though $\left[a, a^\dagger\right] = 1$, $: aa^\dagger :=: a^\dagger a :$.
(2) The symbol : : which is within another symbol : : can be deleted.
(3) A normally ordered product can be integrated or differentiated with respect to a c number, provided the integration is convergent.
(4) The vacuum state projection operator in normal ordering form is

$$|0\rangle\langle 0| =: \exp(-a^\dagger a) : . \tag{18}$$

As a demonstration of IWOP, we use the mathematical formula

$$\int \frac{d^2 z}{\pi} \exp(\tau |z|^2 + \xi z + z^* \eta) = -\frac{1}{\tau} \exp\left(-\frac{\xi\eta}{\tau}\right) \qquad \mathrm{Re}\,\tau < 0 \tag{19}$$

to obtain the overcompleteness relation of the ordinary coherent state $|z\rangle = \exp(-|z|^2/2 + za^\dagger)|0\rangle$:

$$\begin{aligned}\int \frac{d^2 z}{\pi} |z\rangle\langle z| &= \int \frac{d^2 z}{\pi} \exp\left(-\frac{|z|^2}{2} + za^\dagger\right)|0\rangle\langle 0| \exp\left(-\frac{|z|^2}{2} + z^* a\right) \\ &= \int \frac{d^2 z}{\pi} : \exp\left(-|z|^2 + za^\dagger + z^* a - a^\dagger a\right) :=: \exp(a^\dagger a - a^\dagger a) := 1.\end{aligned} \tag{20}$$

The IWOP for Fermi operators was also introduced in [25] and its use in deriving the fermionic Bogoliubov–Valatin transformation was very recently shown in [26]. Now we are dealing with nonlinear Bose operator partners $f(N_a)a$ and $\frac{1}{f(N_a-1)}a^\dagger$, due to (2) and

$$f(N_a)a\frac{1}{f(N_a-1)}a^\dagger = aa^\dagger \qquad \frac{1}{f(N_a-1)}a^\dagger f(N_a)a = a^\dagger a. \tag{21}$$

We see that $f(N_a)a$ and $\frac{1}{f(N_a-1)}a^\dagger$ behave like a and $a^\dagger$, respectively, so we can introduce the generalized normal ordering symbol $\substack{\circ\circ\\\circ\circ}$ for $f(N_a)a$ and $\frac{1}{f(N_a-1)}a^\dagger$. When all $\frac{1}{f(N_a-1)}a^\dagger$ stand on the left of $f(N_a)a$, we say that they are in generalized normal ordering. Correspondingly, the generalized IWOP technique for them can be introduced whose main properties are:

(1) The order of nonlinear Bose operators $f(N_a)a$ and $\frac{1}{f(N_a-1)}a^\dagger$ within a generalized normally ordered product $\substack{\circ\circ\\\circ\circ}$ can be permuted. That is to say, even though $[f(N_a)a, \frac{1}{f(N_a-1)}a^\dagger] = 1$, we have

$$\begin{aligned}\substack{\circ\\\circ} f(N_a)a\frac{1}{f(N_a-1)}a^\dagger \substack{\circ\\\circ} &= \substack{\circ\\\circ} \frac{1}{f(N_a-1)}a^\dagger f(N_a)a \substack{\circ\\\circ} \\ &= \frac{1}{f(N_a-1)}a^\dagger f(N_a)a.\end{aligned} \tag{22}$$

(2) The symbol $\substack{\circ\circ\\\circ\circ}$. which is within another symbol $\substack{\circ\circ\\\circ\circ}$, can be deleted.
(3) A generalized normally ordered product can be integrated or differentiated with respect to a c number, provided the integration is convergent.
(4) The vacuum state projection operator in the generalized normal ordering form is

$$|0\rangle\langle 0| = \substack{\circ\\\circ} \exp\left(-\frac{1}{f(N_a-1)}a^\dagger f(N_a)a\right) \substack{\circ\\\circ}. \tag{23}$$

To prove this we turn the usual normal ordering (18) of $|0\rangle\langle 0|$ to generalized normal ordering, using property (2). We see

$$:(a^{\dagger}a)^n := a^{\dagger n}a^n = \left[\frac{1}{f(N_a-1)}a^{\dagger}\right]^n [f(N_a)a]^n = {}^{\circ}_{\circ}\left[\frac{1}{f(N_a-1)}a^{\dagger}f(N_a)a\right]^n {}^{\circ}_{\circ} \tag{24}$$

and hence

$$|0\rangle\langle 0| = \sum_{n=0}^{\infty}\frac{(-)^n}{n!}\,{}^{\circ}_{\circ}\left[\frac{1}{f(N_a-1)}a^{\dagger}f(N_a)a\right]^n{}^{\circ}_{\circ} = {}^{\circ}_{\circ}\exp\left(-\frac{1}{f(N_a-1)}a^{\dagger}f(N_a)a\right){}^{\circ}_{\circ}. \tag{25}$$

4. Overcompleteness relation of the nonlinear entangled state

To make up a completeness relation of nonlinear pair coherent states we should also introduce the following state:

$$|\eta\rangle\rangle_{f,g} = \exp[-r^2/2 - f(N_a-1)g(N_b-1)a^{\dagger}b^{\dagger} + \eta f(N_a-1)a^{\dagger} + \eta^* g(N_b-1)b^{\dagger}]\,|00\rangle\,. \tag{26}$$

By using the generalized IWOP technique and

$$|00\rangle\langle 00| = {}^{\circ}_{\circ}\exp\left[-\frac{1}{f(N_a-1)}a^{\dagger}f(N_a)a - \frac{1}{g(N_b-1)}b^{\dagger}g(N_b)b\right]{}^{\circ}_{\circ}$$

we can easily verify that $|\eta\rangle_{f,g}$ and $\left(|\eta\rangle\rangle_{f,g}\right)^{\dagger}$ satisfy the overcompleteness relation

$$\begin{aligned}
\int\frac{d^2\eta}{\pi}\,|\eta\rangle_{f,g}\;{}_{f,g}\langle\langle\eta| &= \int\frac{d^2\eta}{\pi}\exp\bigg[-|\eta|^2/2 - \frac{1}{f(N_a-1)g(N_b-1)}a^{\dagger}b^{\dagger} \\
&\quad + \frac{\eta}{f(N_a-1)}a^{\dagger} + \frac{\eta^*}{g(N_b-1)}b^{\dagger}\bigg] \\
|00\rangle\langle 00|\exp&\left[-|\eta|^2/2 - abf(N_a-1)g(N_b-1) + \eta^* af(N_a-1) + \eta bg(N_b-1)\right] \\
&= \int\frac{d^2\eta}{\pi}\,{}^{\circ}_{\circ}\exp\bigg[-|\eta|^2 - \frac{1}{f(N_a-1)g(N_b-1)}a^{\dagger}b^{\dagger} \\
&\quad + \eta\left(\frac{1}{f(N_a-1)}a^{\dagger} + bg(N_b-1)\right) + \eta^*\left(\frac{1}{g(N_b-1)}b^{\dagger} + af(N_a-1)\right) \\
&\quad - \frac{1}{f(N_a-1)}a^{\dagger}f(N_a)a - \frac{1}{g(N_b-1)}b^{\dagger}g(N_b)b \\
&\quad - \frac{1}{f(N_a-1)g(N_b-1)}a^{\dagger}b^{\dagger} - abf(N_a-1)g(N_b-1)\bigg]{}^{\circ}_{\circ} = 1
\end{aligned} \tag{27}$$

where we have use equation (19). It can be rewritten as

$$\frac{1}{\pi}\int_0^{\infty} r\,dr\int_0^{2\pi} d\varphi\,\left|\eta = re^{-i\varphi}\right\rangle_{f,g}\;{}_{f,g}\left\langle\left\langle\eta = re^{-i\varphi}\right|\right. = 1. \tag{28}$$

Then we introduce

$$\left|\left|r^2, q\right\rangle\right\rangle_{f,g} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{iq\varphi}\,\left|\eta = re^{-i\varphi}\right\rangle\rangle_{f,g}\,. \tag{29}$$

By using equations (14) and (29), we calculate

$$\sum_{q=-\infty}^{+\infty}\int_0^\infty \mathrm{d}(r^2)||r^2,q\rangle_{f,g}\ {}_{f,g}\langle\langle r^2,q|| = 1 \tag{30}$$

then the orthonormal property of $\left||r^2,q\right\rangle_{f,g}$ and $\left||r^2,q\right\rangle\rangle_{f,g}$ is obvious:

$$\begin{aligned}&{}_{f,g}\left\langle\langle r'^2,q'\right||\left[f(N_a)a+\frac{1}{g(N_b-1)}b^\dagger\right]\left[g(N_b)b+\frac{1}{f(N_a-1)}a^\dagger\right]\left||r^2,q\right\rangle_{f,g}\\&\quad = r'^2_{f,g}\left\langle\langle r'^2,q'\right||\,r^2,q\right\rangle_{f,g} = r^2_{f,g}\left\langle\langle r'^2,q'\right||\,r^2,q\right\rangle_{f,g}\end{aligned} \tag{31}$$

$${}_{f,g}\left\langle\langle r'^2,q'\right||\,(N_a-N_b)\left||r^2,q\right\rangle_{f,g} = q'_{f,g}\left\langle\langle r'^2,q'\right||\,r^2,q\right\rangle_{f,g} = q_{f,g}\left\langle\langle r'^2,q'\right||\,r^2,q\right\rangle_{f,g} \tag{32}$$

and

$${}_{f,g}\left\langle\langle r'^2,q'\right||\,r^2,q\right\rangle_{f,g} = \frac{1}{2r}\delta(r-r')\delta_{q,q'}. \tag{33}$$

5. Two-mode nonlinear squeezing transformation

In a nonlinear entangled state representation we can calculate the following bra-ket integration:

$$\begin{aligned}S_{f,g}(\lambda) &= \int\frac{\mathrm{d}^2\eta}{\pi}|\eta/u\rangle_{f,g}\ {}_{f,g}\langle\langle\eta| = {}^{\circ}_{\circ}\int\frac{\mathrm{d}^2\eta}{\pi}\exp\left[-\frac{|\eta|^2}{2}\left(1+\frac{1}{u^2}\right)\right.\\&\quad+\frac{\eta}{f(N_a-1)u}a^\dagger+\frac{\eta^*}{g(N_b-1)}b^\dagger-\frac{1}{f(N_a-1)g(N_b-1)}a^\dagger b^\dagger\\&\quad-\frac{1}{f(N_a-1)}a^\dagger f(N_a)a-\frac{1}{g(N_b-1)}b^\dagger g(N_b)b-abf(N_a-1)g(N_b-1)\\&\quad\left.+\eta^* af(N_a-1)+\eta bg(N_b-1)\right]{}^{\circ}_{\circ}\\&= (\sec h\lambda)\,{}^{\circ}_{\circ}\exp\left[\tanh\lambda\frac{1}{f(N_a-1)}a^\dagger\frac{1}{g(N_b-1)}b^\dagger\right.\\&\quad+\left(\frac{1}{f(N_a-1)}a^\dagger f(N_a)a+\frac{1}{g(N_b-1)}b^\dagger g(N_b)b\right)(\sec h\lambda-1)\\&\quad\left.-\tanh\lambda f(N_a)ag(N_b)b\right]{}^{\circ}_{\circ} = \exp\left[\tanh\lambda\frac{1}{f(N_a-1)}a^\dagger\frac{1}{g(N_b-1)}b^\dagger\right]\\&\quad\times\exp\left[\left(a^\dagger a+b^\dagger b+1\right)\ln\sec h\lambda\right]\cdot\exp\left[-\tanh\lambda f(N_a)ag(N_b)b\right]\end{aligned} \tag{34}$$

where $u = \mathrm{e}^\lambda$, and in the last step we have used the operator identity

$$\mathrm{e}^{ka^\dagger a} =: \exp\{(\mathrm{e}^k-1)a^\dagger a\} := {}^{\circ}_{\circ}\exp\left\{(\mathrm{e}^k-1)\frac{1}{f(N_a-1)}a^\dagger f(N_a)a\right\}{}^{\circ}_{\circ}. \tag{35}$$

Under the $S_{f,g}\,(\lambda)$ transformation,

$$S_{f,g}\,(\lambda)\,\frac{1}{f(N_a-1)}a^\dagger S_{f,g}\,(\lambda)^{-1} = \frac{1}{f(N_a-1)}a^\dagger\cosh\lambda - g(N_b)b\sinh\lambda \tag{36}$$

$$S_{f,g}\,(\lambda)\,f(N_a)aS_{f,g}\,(\lambda)^{-1} = f(N_a)a\cosh\lambda - \frac{1}{g(N_b-1)}b^\dagger\sinh\lambda \tag{37}$$

which is a two-mode nonlinear squeezing transformation. Moreover, from $S_{f,g}$ we can in turn derive

$$\begin{aligned} S_{f,g}(\lambda)|q,0\rangle &= (\sec h\lambda)^{q+1}\exp\left[\tanh\lambda\frac{1}{f(N_a-1)}a^\dagger\frac{1}{g(N_b-1)}b^\dagger\right]|q,0\rangle \\ &= (\sec h\lambda)^{q+1}\sum_{n=0}^{\infty}\frac{[(n+q)!]^{1/2}(\tanh\lambda)^n}{(n!q!)^{1/2}\prod_{l=q}^{n+q-1}f(l)\prod_{m=0}^{n-1}g(m)}|n+q,n\rangle. \end{aligned}$$

When $f = g = 1$, $S_{f,g}(\lambda)$ becomes the well-known two-mode squeezing operators [27], and

$$S_{f,g}(\lambda)|q,0\rangle \to (\sec h\lambda)^{q+1}\sum_{n=0}^{\infty}\frac{[(n+q)!]^{1/2}(\tanh\lambda)^n}{(n!q!)^{1/2}}|n+q,n\rangle$$

which is a nonlinear charge-conserved state with negative binomial distribution [16,28], since the modulus square of the coefficient of $|n+q,n\rangle$ gives

$$(\sec h^2\lambda)^{q+1}\frac{(n+q)!}{n!q!}\left(\tanh^2\lambda\right)^n.$$

In summary, based on two-mode NCS we have generalized BBDR conserved-charge coherent states to the nonlinear case, and a new nonlinear entangled state $|\eta\rangle_{f,g}$ as well as its deductive state are also derived. Their completeness relation is proved by virtue of the generalized IWOP technique. A new two-mode nonlinear squeezing operator in the $|\eta\rangle_{f,g}$ representation is obtained, which in turn leads to a nonlinear negative binomial state.

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